directly above one another can be significant. In the case mentioned above, the temperatures increased by more than 10% with respect to the case of no interaction, i.e. one device only.

The numerical results of cases 3 and 4 were obtained with a mapping function for which $\Delta X_{\min}$ and C were equal to 0.05 and 1.125, respectively, as defined in ref. [5]. An 84 × 21 grid was employed such that nodes 31 through 39 represented the heat source. Minimum ΔX spacings where fixed between nodes 25 and 45. These numerical results were obtained with the following transient coefficients and time step: $\alpha_\omega = 0.01$, $\alpha_\psi = 1.0$, $\alpha_\Theta = 1.0$ and $\Delta\tau = 0.02$. The computations required about 1.2 CPU hours on Bell Laboratories IBM 370/168 computer. These results were obtained with significantly less computing time than the results of case 1 (2 CPU hours) [5]. Although a larger number of nodes were used in the present study, the new mapping function resulted in a smaller overall duct length. The time step used for the new results was twice that used in ref. [5]. Earlier experimentation with this code showed that heat flux boundary specification significantly increased the necessary computing times, as compared with constant wall temperature boundary conditions. The overall duct length was also observed to play a pronounced role in determining the time for convergence.

For future work, the strong effect of axial conduction found experimentally in these local source problems requires the rigorous addition of axial conduction into the numerical solution. Another problem of interest would be to study the effects on heat transfer of a strong heat source upstream of an equivalent or weaker source along the same wall, and the resulting wake interaction.

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FREQUENCY DEPENDENCE IN THE EQUIVALENT DIFFUSIVITY

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NOMENCLATURE

Greek symbols

- ω, frequency of harmonic excitation;
- λ, eigenvalue in Fourier method;
- ν, dimensionless frequency parameter;
- ξ, eigenvalue;
- κ, (equivalent) diffusivity;
- α, coefficients in low frequency series expansion of ξ.

Subscripts

- M, refers to matrix layer;
- F, refers to filler layer;
- eq, refers to equivalent property.

1. INTRODUCTION

THE PROBLEM of determining the temperature distribution in bilaminates which are stacked parallel to an harmonically excited heat flow of frequency ω (Fig. 1), using effective thermal properties, has drawn considerable attention in recent years. In the case of a steady exciting heat source, it is well known that a single parameter, the equivalent diffusivity η_{eq}, will characterize the heat flow. Even for composites of materials with similar properties, the equivalent diffusivity will be adequate for the non-steady source of any low frequency. On the other hand, if ω is not vanishingly small, or if the materials differ greatly, then it has been suggested [5] that η_{eq} should be replaced by two equivalent diffusivities, η_P and η_A, diffusivity for phase and amplitude respectively, each dependent on the frequency. An experimental study by Truong and Zinsmeister [6] confirmed the failure of the static equivalent diffusivity and demonstrated that, in the cases they considered, η_P was roughly equal to η_{eq}, whereas η_A was less than η_{eq}. These results were the more obvious in composites of very unlike material. Difficulties inherent in determining experimental values of η_A and η_P at high exciting frequencies highlight the importance of an analytic solution of the heat equation that will yield explicit values for η_A and η_P.

We present an analytic attack of the problem of determining these effective thermal constants. We find it more convenient to use a dimensionless frequency parameter ν. Expanding the work of Horvay [5] to develop a frequency dependent η_{eq} allows explicit results for η_A and η_P. Numerical refinements of η_{eq} using Pade approximants and Newton's method, as well as

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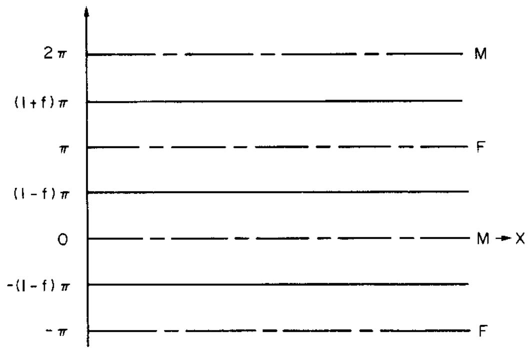

FIG. 1. A 2-layered laminated composite of type M–F–M.

an alternate expansion in the case of extremely high ν, are examined.

Consulting Fig. 1, where matrix layers, labelled M, and filler layers, labelled F, with volume fraction f, are stacked, we search for the temperature distribution in the composite,

$$T = \begin{cases} T_M & 0 \leqslant y \leqslant (1-f)\pi, \\ T_F & (1-f)\pi \leqslant y \leqslant \pi. \end{cases} \tag{1}$$

The conventional approach is to take η_{eq} as a weighted average of the constituent layers

$$\eta_{eq} = k_{eq}/[\rho c]_{eq} = \frac{(1-f)k_M + f k_F}{(1-f)(\rho c)_M + f(\rho c)_F} \tag{2}$$

where f, k, and ρc are the volume fraction of the filler layer, thermal conductivity and the heat capacity, respectively. Clearly equation (2) does not represent a frequency dependent η_{eq}.

Temperature in the composite is governed by the heat equation (3), where T is a function of position and time

$$\left(\nabla^2 - \frac{1}{\eta_J}\frac{\partial}{\partial t}\right)T_{J=0}, \quad J = \mathrm{M,F}. \tag{3}$$

The restriction that only harmonic excitations will be considered allows equation (3) to be written as

$$\left(\nabla^2 - \mathrm{i}\frac{\omega}{\eta_J}\right)v_J = 0, \quad J = \mathrm{M,F} \tag{4}$$

where i indicates a phase shift. Equation (4) is subject to the boundary conditions (5)

$$\begin{aligned} \text{at} \quad & y = 0: \ \partial T/\partial y = 0, \\ & y = (1-f)\pi: \ T_M = T_F \text{ and } (\partial T_M/\partial y)/\kappa_M \\ & \quad = (\partial T_F/\partial y)/\kappa_F, \\ & x = 0: \ T = \exp(-\mathrm{i}\omega t)\chi(y), \\ & x = \infty: \ T = 0. \end{aligned} \tag{5}$$

The solution of equation (4) subject to conditions (5) in the form of products

$$\begin{aligned} w_M &= \exp(-\lambda x)\psi_M(y) \\ w_F &= \exp(-\lambda x)\psi_F(y) \end{aligned} \tag{6}$$

where $\lambda = \lambda_R + \mathrm{i}\lambda_I$, was examined in detail by Horvay [5].

It was found that

$$\lambda^2 = z^2/(1-f)^2 - \mathrm{i}\omega/\eta_M = z'^2/(1-f)^2 - \mathrm{i}\omega/\eta_F. \tag{7}$$

A dimensionless frequency parameter ν was introduced

$$\nu = \omega(1-f)^2(1/\eta_F - 1/\eta_M). \tag{8}$$

The solution of equation (4) leads to an eigenvalue equation (9) in which z and z' are the roots

$$\begin{aligned} 0 &= \pi\Delta/a = J \tan J + b/aJ' \tan aJ', \\ J &= z\pi, \quad J' = z'\pi, \\ a &= f/(1-f), \quad b = a\kappa_F/\kappa_M. \end{aligned} \tag{9}$$

2. LOW FREQUENCY ESTIMATE OF THE ROOTS

We expand J in whole powers of ν to obtain the roots of the eigenvalue equation (9) which can then be used to solve η_{eq},

$$\begin{aligned} J^2 &= (z\pi)^2 = \mathrm{i}\nu\pi^2\alpha_1 + \nu^2\pi^4\alpha_2 + \mathrm{i}\nu^3\pi^6\alpha_3 + \ldots \\ (J')^2 &= (z'\pi)^2 = (z^2 + \mathrm{i}\nu)\pi^2 = \mathrm{i}\nu\pi^2\beta_1 + \nu^2\pi^4\alpha_2 + \ldots \end{aligned} \tag{10}$$

The eigenvalue equation may be represented as a series as well

$$\begin{aligned} 0 &= J \tan J + b/aJ' \tan aJ' \\ &= J^2 + b(J')^2 + 1/3[J^4 + ba^2(J')^4] \\ &\quad + 2/15(J^6 + ba^4(J')^6) + \ldots \end{aligned} \tag{11}$$

Using the notation of ref. [5],

$$\begin{aligned} \kappa'_M &= (1-f)\kappa_M; \quad \tilde{\kappa}_M = f\kappa_M, \\ \kappa'_F &= f\kappa_F; \quad \tilde{\kappa}_F = (1-f)\kappa_F, \\ \kappa_{av} &= \kappa'_M + \kappa'_F\tilde{\kappa} = \tilde{\kappa}_M + \tilde{\kappa}_F, \\ \beta_1 &= 1 + \alpha_1. \end{aligned} \tag{12}$$

The expressions in equations (10) are substituted into equation (11), and we solve for the α's by equating the coefficients of successive powers of ν to 0. To the 1st order,

$$0 = \mathrm{i}\nu\pi^2\alpha_1 + b\mathrm{i}\nu\pi^2(1+\alpha_1),$$

so

$$\alpha_1 = -b/(1+b) = -\kappa'_F/\kappa_{av}. \tag{13}$$

Similarly, for higher orders of ν we find

$$\begin{aligned} \alpha_2 &= \frac{1}{3}(\alpha_1^2 + ba^2\beta^2)/(1+b), \\ &= \frac{1}{3}\kappa'_F\tilde{\kappa}_M\tilde{\kappa}/\kappa_{av}^3 \end{aligned} \tag{14}$$

$$\alpha_3 = \frac{2}{15}(\alpha_1^3 + ba^4\beta^3) - \frac{2}{3}(\alpha_1\alpha_2 + ba^2\beta\alpha_2)/(1+b),$$

$$\begin{aligned} \alpha_4 = &-\frac{272}{7!}(\alpha_1^4 + ba^4\beta^4) + \frac{6}{15}(\alpha_1^2\alpha_2 + ba^4\beta^2\alpha_2) \\ &- \frac{1}{3}(\alpha_2^2 - \alpha_1\alpha_3 + ba^2(\alpha_2^2 - 2\beta\alpha_3))/(1+b). \end{aligned}$$

Determination of the α's uniquely determines values of the roots z and z' which may be manipulated to solve for η_{eq},

$$\begin{aligned} -\mathrm{i}\omega/\eta_{eq} &= \lambda^2 = -\mathrm{i}\omega/\eta_M + z^2/(1-f)^2, \\ &= -\mathrm{i}\omega/\eta_M + (\mathrm{i}\nu\pi^2\alpha_1 + \nu^2\pi^4\alpha_2 + \mathrm{i}\nu^3\pi^6\alpha_3 + \ldots)/(1-f)^2\pi^2. \end{aligned} \tag{15}$$

Therefore,

$$[\eta_{eq}]^{-1} = \left(1/\eta_M - \frac{\mathrm{i}\nu\pi^2\alpha_1 + \nu^2\pi^4\alpha_2 + \mathrm{i}\nu\pi^6\alpha_3 + \ldots}{\mathrm{i}(1-f)^2\pi^2\omega}\right).$$

Approximations of η_{eq}^{-1} to successive orders of ν are listed below: to order 0:

$$[\eta_{eq}]^{-1} = \eta_M^{-1} + \kappa'_F/\kappa_{av}(\eta_F^{-1} - \eta_M^{-1}); \tag{16}$$

and recursively from there by

$$[\eta_{eq}]^{-1}_{\text{order } k+1} = [\eta_{eq}]^{-1}_{\text{order } k} - [\mathrm{i}\omega\pi^2(1-f)^2]^k\left(\frac{1}{\eta_F} - \frac{1}{\eta_M}\right)^{k+1}\alpha_{k+2}V$$

where

$$V = \begin{cases} 1 \text{ if } k \text{ is even}, \\ -\mathrm{i} \text{ if } k \text{ is odd}. \end{cases} \tag{17}$$

Table 1. η_{eq} via series, Pade [1/1] and approximants, and Newton's method (η_{eq} written: real, imaginary)

v	Series	[1/1]	[2/2]	[Newton]
$f = 0.3$				
0.00001	3.28347, -4.0×10^{-6}	3.28347, -4.0×10^{-6}	3.28347, -4.0×10^{-6}	3.28347, 4.0×10^{-6}
0.01	3.2835, −0.004	3.2835, −0.004	3.2835, −0.004	3.28352, 0.008
0.1	3.28588, −0.03985	3.28588, −0.03985	3.28588, −0.03985	3.28802, 0.07978
1	3.41041, −0.20218	3.46045, −0.29312	3.45867, −0.29243	3.65822, 0.61710
10	0.00686, −0.00093	3.92823, −0.10678	3.95041, −0.14877	3.98226, 0.19567
$f = 0.6$				
0.00001	2.18679, -7.5×10^{-5}	2.18679, -7.5×10^{-5}	2.18679, -7.5×10^{-5}	2.18679, -7.5×10^{-5}
0.01	2.19084, −0.07522	2.19084, −0.07523	2.19084, −0.07523	2.19139, 0.15089
0.1	2.31179, −0.17753	2.5016, −0.58515	2.49581, −0.58286	2.68678, 1.4764
1	0.00089, 9.9×10^{-5}	3.54242, −0.25198	3.62423, −0.34588	3.49442, −0. 21755
10	8.9×10^{-8}, 9.9×10^{-10}	3.58877, −0.02606	3.73418, −0.03922	4.06417, 0.23781
$f = 0.9$				
0.00001	0.66163, −0.00144	0.66163, −0.00144	0.66163, −0.00144	0.66163, 0.00287
0.01	0.00985, 0.00302	1.37600, −0.79132	1.32217, −0.85697	4.36358, 0.76503
0.1	9.8×15^{-7}, 3.1×15^{-8}	2.23328, −0.17409	2.44883, −0.2392	2.64974, −0.70177
1	9.9×10^{-11}, 3.1×10^{-13}	2.25237, −0.01762	2.48117, −0.02436	3.91616, −0.72066
10	9.9×10^{-5}, 3.09×10^{-18}	2.25286, −0.00126	2.48149, −0.00244	3.88043, −0.00043

Note that the 0th order approximation is identical to the static η_{eq} in equation (2), as expected.

A [1/1] and [2/2] Pade approximant was used to refine the series representation of η_{eq}. The Pade adjusted η_{eq} was then used as a first guess in Newton's iterative method to solve the eigenvalue equation (9). A computer program was developed to calculate the roots to within a tolerance of 10^{-5}. These roots were then back-substituted into equation (15) to obtain a Newtonized η_{eq} value. Results of the series, Pade, and Newtonized approximations are summarized in Tables 1 and 2.

3. THE HIGH FREQUENCY EXPANSION

The series representation for η_{eq} developed in equations (16)–(20) diverges rapidly for $\omega \gg 1$, or for η_M and $\eta_F \ll 1$. To overcome this problem we expand J as in ref. [4],

$$J = G_{1/2}v^{1/2} + G_0 v^0 + G_{-1/2}v^{-1/2} + G_{-1}v^{-1} + \ldots \quad (18)$$

We find that

$$G_{1/2} = \mathrm{i}^{-1/2}\pi,$$

$$G_0 = 0,$$

$$G_{-1/2} = \mathrm{i}^{1/2}\pi(1-f)^2/8f^2, \quad (19)$$

$$G_{-1} = -\kappa_F(1-f)^2/(4\kappa_M f^3),$$

$$G_{-3/2} = \mathrm{i}^{-1/2}[3(\kappa_F/\kappa_M)^2 + \pi^2/16](1-f)^4/(8\pi f^4).$$

A derivation similar to that used in Section 2 leads to an explicit representation for η_{eq} as

$$[\eta_{eq}]^{-1} = \left[1/\eta_F + \frac{\mathrm{i}}{4f^2\omega} - \frac{(2\kappa_F)^{1/2}}{4\kappa_M f^3 Q^{1/2}\omega^{3/2}} - \frac{(\sqrt{2})\kappa_F \mathrm{i}}{4\kappa_M f^3 Q^{1/2}\omega^{3/2}} + 3\kappa f^2/4\pi^2 f^4 \kappa_M^2 \omega^2 Q \right] \quad (20)$$

where

$$Q = (1/\eta_F - 1/\eta_M).$$

Table 2. η_{eq} via series, Pade [1/1], Pade [2/2] and Newton, $\eta_\mu = 1.71$, $\eta_F = 1.18$ (η_{eq} written: real, imaginary)

v	Series	[1/1]	[2/2]	Newton
$f = 0.3$				
0.00001	1.54940, -1.37×10^{-6}	1.54940, -1.37×10^{-6}	1.54940, -1.32×10^{-6}	1.54940, 2.079×10^{-6}
0.01	1.54941, −0.00137	1.54941, −0.00137	1.54941, −0.00127	1.54940, 0.00274
0.1	1.55004, −0.01370	1.55010, −0.01367	1.55009, −0.01368	1.54914, 0.02735
1	1.13339, −0.06841	1.60497, −0.10857	1.60393, −0.11016	1.53911, 0.23573
10	0.00037, 4.80×10^{-7}	1.80727, −0.05039	1.81567, −0.05380	0.79008, −0.78614
$f = 0.6$				
0.00001	1.39018, -6.72×10^{-6}	1.39018, -6.72×10^{-6}	1.39008, -6.72×10^{-6}	1.39018, 0.01343
0.01	1.39017, −0.00672	1.39017, 0.00672	1.39017, −0.00672	1.39026, 0.01343
0.1	1.39465, −0.07324	1.38866, −0.06713	1.39144, −0.06957	1.39847, 0.13383
1	−0.02981, −0.00282	1.24508, −0.63872	1.67208, −0.57094	1.17378, −0.67164
10	-2.95×10^{-6}, -2.44×10^{-6}	−1.08659, −1.09027	2.74525, −0.26658	2.88309, −0.41722
$f = 0.9$				
0.00001	1.23232, -1.865×10^{-5}	1.23232, -1.86×10^{-5}	1.23232, 1.86×10^{-5}	1.23232, 3.7×10^{-5}
0.01	1.23848, −0.01565	1.22822, −0.01759	1.22888, −0.01769	1.23600, 0.04012
0.1	−0.01488, 0.00048	1.16515, −0.02878	1.15710, −0.03922	1.38217, 0.09497
1	-1.47×10^{-6}, 4.89×10^{-9}	1.15297, −0.00340	1.13689, −0.00498	1.17452, −0.50306
10	-1.47×15^{-10}, 4.89×10^{-14}	1.15282, −0.00034	1.13663, −0.0005	1.50542, −0.18970

Table 3. η_A and η_P via Pade [2/2] and Newton when $\eta_\mu = 4.11$, $\eta_F = 0.0116$

v	$\eta_A[2/2]$	$\eta_P[2/2]$	η_A	η_P
$f = 0.3$				
0.00001	3.28347	3.28348	3.28347	3.28348
0.01	3.27950	3.28750	3.29155	3.27555
0.1	3.24675	3.32646	3.37075	3.21110
1	3.20131	3.79034	4.45012	3.18081
10	3.80983	4.1078	3.80055	4.19283
$f = 0.6$				
0.00001	2.18672	2.18687	2.18672	2.18687
0.01	2.1194	2.27003	2.3586	2.05539
0.1	2.08810	3.3174	5.9136	2.06918
1	3.32483	4.02289	3.29636	3.73315
10	3.69558	3.77402	4.32368	3.84644
$f = 0.9$				
0.00001	0.6602	0.66307	0.66452	0.65877
0.01	1.02421	3.45276	3.7777	5.35486
0.1	2.24248	2.72544	2.18237	3.68436
1	2.45716	2.50589	4.86183	3.3717
10	2.47906	2.48394	3.88000	3.88086

Table 4. η_A and η_P via Pade [2/2] and Newton when $\eta_\mu = 1.71$, $\eta_F = 1.18$

v	$\eta_A[2/2]$	$\eta_P[2/2]$	η_A	η_P
$f = 0.3$				
0.00001	1.54940	1.54940	1.54941	1.54940
0.01	1.54804	1.44078	1.55215	1.54667
0.1	1.53659	1.56395	1.57723	1.52250
1	1.50461	1.72598	1.83485	1.35232
10	1.76421	1.87191	0.65387	3.78083
$f = 0.6$				
0.00001	1.39018	1.39019	1.3902	1.39017
0.01	1.38349	1.39694	1.40389	1.37703
0.1	1.32691	1.46641	1.55278	1.28267
1	1.33982	2.61404	0.90328	2.68666
10	2.51508	3.05327	2.54814	3.40014
$f = 0.9$				
0.00001	1.23230	1.23234	1.23236	1.23228
0.01	1.21156	1.24695	1.27812	1.19779
0.1	1.11983	1.19837	1.48738	1.29655
1	1.13195	1.14190	0.91677	2.10745
10	1.13614	1.13713	1.34870	1.73414

4. THE PHASE AND AMPLITUDE SEPARATION

The solution of equation (4) subject to $T(0, y, t) = \cos \omega t$ may be written as

$$T(x, y, t) = \exp\left[-x(\omega/2\eta_{eq})^{1/2}\right] \cos\left[\omega t - x(\omega/2\eta_{eq})^{1/2}\right] \quad (21)$$

where η_{eq} is allowed to be complex. If we instead write

$$T(x, y, t) = \exp\left[-x(\omega/2\eta_A)^{1/2}\right] \cos\left[\omega t - x(\omega/2\eta_P)^{1/2}\right] \quad (22)$$

with η_A and η_P real, then we can relate η_A, η_P to η_{eq}. For example, we know that

$$\lambda = (\omega/2\eta_{eq})^{1/2}(1-\mathrm{i}) \quad (23)$$

whereas equation (22) leads to

$$\lambda = (\omega/2\eta_A)^{1/2} - \mathrm{i}(\omega/2\eta_P)^{1/2}. \quad (24)$$

Since

$$\frac{\lambda^2}{\omega} = \eta_A - 1 - \eta_P^{-1} - 2\mathrm{i}\left(\frac{1}{\eta_A \eta_P}\right)^{1/2} = \frac{2\mathrm{i}}{\eta_{eq}}, \quad (25)$$

if we write $\eta_{eq}^{-1} = R + \mathrm{i}I$ we get the two equations

$$\eta_A^{-1} - \eta_P^{-1} = 2I, \quad (26a, b)$$

$$(\eta_A \eta_P)^{-1} = R^2.$$

For example, if η_{eq} is real then $\eta_A = \eta_P = \eta_{eq}$, and this is experimentally known to be false unless $v \ll 1$.

We can in fact solve the system exactly to get the following results:

$$[\eta_A]^{-1} = +I + (I^2 + R^2)^{1/2},$$
$$[\eta_P]^{-1} = -I + (I^2 + R^2)^{1/2}. \quad (27a, b)$$

Numerical results using I and R given by the Newtonized values of η_{eq} are used directly in equations (27), and are summarized in Tables 3 and 4. The values derived in this manner agree substantially with those found by Truong and Zinsmeister [6], the main difference being a tendency on the part of the Newtonized values to jump around, whereas the values from the Pade approximation do demonstrate the expected behavior. One explanation of the strange behavior from certain Newtonized values is that the method might be picking up higher order eigenvalues. In the analytic approach here, it is clear that as $\omega \to 0$, both η_A and η_P approach η_{eq}. Both the analytic and experimental approach clearly demonstrate that the static equivalent diffusivity is unreliable for non-vanishingly small values of v. In short, the double-diffusivity model is a significant improvement over the static η_{eq} model. Since the quantities η_F, η_M, κ_F, κ_M, f are readily obtained, there is no practical reason to insist upon the static equivalent.

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